SIMULATION OF SURFACE GENERATION BY THE MIXED DIFFRACTION-FINITE ELEMENT MODEL

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Abstract

The displacement measurement technique based on the generation of surface waves is analysed. The simulation technique have been developed for the estimation of the efficiency of surface waves generation.

The technique is based on two different models: the diffraction model and the finite element model. During the first stage of analysis the diffraction model has been used in order to calculate the spatial distribution and the time shape of the acoustic pressure on the plane corresponding to the boundary of the object where surface waves have to be generated. During the second stage, the propagation of acoustic waves in the object is simulated by means of the finite element technique. The excitation forces applied on the finite element model are obtained by using the acoustic pressure calculated from the diffraction model. Finally the efficiency of the transducer is estimated analyzing the simulation results.

The proposed approach enables to analyze the efficiency of the transducers of different design including multi-element transducer arrays. The generation of the surface waves in objects with non-planar boundaries can be investigated by this technique as well.

Introduction

The ultrasonic distance or displacement measurements are used in many industrial applications. They are based on measurement of the ultrasonic wave propagation time between the transducer and the object distance to which has to be measured. The disadvantage of such an approach is that the ultrasonic wave propagation time depends not only upon the distance, but also upon the ultrasound velocity in the medium (gas or liquid) between the object and the transducer. In both cases the ultrasound velocity depends on properties of the medium including temperature, humidity, etc. It is a source of errors and requires complicated compensation algorithms when high accuracy is necessary. Therefore other techniques free from such disadvantage are desirable. It was proposed to use the approach based on the generation surface or plate waves for the displacement measurement. The main objective of presented research was to develop the simulation technique enabling to investigate the possibilities of such measurements.

Set up of the displacement measurement technique

The possible solution of the approach is explained in Figure 1.

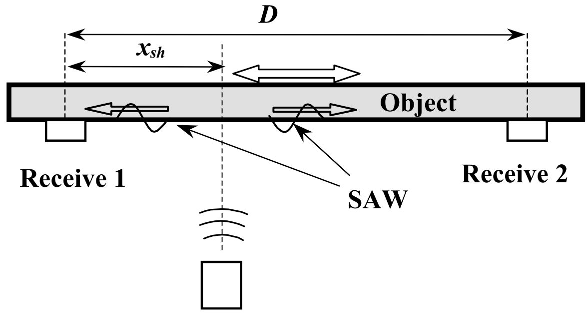

Figure 1: Displacement measurement technique using surface waves

The ultrasonic transducer in this case is situated at some distance from the moving surface and oriented perpendicularly to the surface. The ultrasonic waves generated by the transmitter propagate through the contacting medium and hit the surface of the object. On the boundary of the object the sophisticated transformation of the wave occurs. Partially the waves are transmitted/refracted inside the object as longitudinal or shear waves. Simultaneously the surface waves are generated due to the limited dimensions of the ultrasonic beam. The surface waves propagate to opposite directions and can be received by two receivers situated on both sides of the surface of the object. The distance D between the receivers is precisely known. The delay times t_1, t_2 of signals received by each of receiver can by expressed as

$$t_1 = L / c_m + x_{sh} / c_s, \quad t_2 = L / c_m + \left(D - x_{sh}\right) / c_s, \tag{1}$$

where L is the distance between the transmitter and the object, c_m and c_s represent the ultrasound velocity in the surrounding medium and of the surface waves on the object correspondingly, x_{sh} is the distance of the displacement of which has to be be measured. It can be easily demonstrated that displacement distance can be expressed by

$$x_{sh} = \frac{D - c_m \cdot \left(t_2 - t_2\right)}{2}. \tag{2}$$

So the measured displacement value is proportional only to the delay time difference and does not depend on the ultrasound velocity in the surrounding medium.

Simulation approach

The simulation of the complete acoustic part of the considered system is very complicated as it must take into account the spatial distribution of the acoustic waves, as well as, the generation and propagation of the surface waves. Due to the complicated geometry of the wave propagation domain, at least 2D approach has to be used. The most universal technique for the investigation ultrasonic waves propagation in the case of complicated geometry is finite element (FE)_ method. FE simulation of elastic waves in solids enables to obtain the time varying displacement and strain-stress fields over the finite element mesh representing the investigated body. The solutions obtained by the FE method are very close to reality and highly reliable.

The direct implementation of the FE technique requires excessively large computational resources, therefore here we use the mixed approach. The well-known techniques based on the diffraction theory [2-4] enable to calculate the ultrasonic field generated by different transducers. The techniques are approved in many applications. However they give no information on the phenomena taking place at the boundary between two media. The wave transformation on the boundary can be investigated by using the FE analysis.

In the first stage of the mixed approach the structure of the field generated by the transmitter is calculated by using the diffraction-based method. The 3D model allowing to investigate the fields of the arbitrary contour planar transducers has been used [5]. The approach exploits the fact that the pulse response for a velocity potential at arbitrary point *M*(x,y,z) at some time instance *t* is proportional to the arc angle of on the surface of transducer [2-3]. The arc consists of the set of points the distance of which to point *M* is the same. The value of the angle and the velocity potential is obtained by using numerical methods. The method works for any shape of the transducer contour, which can be approximated by some set of line segments. The result of the first stage of analysis is the calculated acoustic pressure distribution $p(x,y,z,t)$ on the surface of the object.

The obtained pressure distribution is used as input data for finite element model defining excitation. The problem of linking of the two models is that diffraction model brings 3D results, and the application of 3D finite element models is too expensive from the point of view of computational resources. In this investigation the transmitter was selected creating relatively uniform distribution of the acoustic field along one of its axes. The pressure distribution on the plane perpendicular to the axis has been used as input data for the 2D finite element model.

The propagation of waves in 2D elastic continua is described by means of the differential equations inside of volume *V* and boundary conditions on its boundary *S* as

$$\begin{aligned} &[\mathbf{A}]^T\{\boldsymbol{\sigma}\}+\{\mathbf{b}\}=\rho\{\ddot{\mathbf{u}}\}, && \in V \\ &\{\mathbf{t}\}=[\mathbf{A}_s]^T\{\boldsymbol{\sigma}\} && \in S \end{aligned} \qquad (3)$$

where $\{\boldsymbol{\sigma}\}$ is stress tensor in Voigt's notation, $\{\mathbf{b}\}$ is body force vector, $\{\mathbf{t}\}$ is tractions vector on surface *S*, $\{\mathbf{u}\}$ is displacement vector of any point of the continua, ρ is density of the material, $[\mathbf{A}]$ is differential operator, $[\mathbf{A}_s]$ is matrix containing components of unit normal vector $\{\mathbf{n}\}$ to surface *S*. In 2D case we have

$$[\mathbf{A}]=\begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix},[\mathbf{A}_s]=\begin{bmatrix} n_x & 0 \\ 0 & n_y \\ n_y & n_x \end{bmatrix}.$$

The constitutive equation of elasticity reads as $\{\boldsymbol{\sigma}\}=[\mathbf{c}]\{\boldsymbol{\varepsilon}\}$, where

$$[\mathbf{c}]=G\begin{bmatrix} 4/3 & -2/3 & 0 \\ -2/3 & 4/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}+K_v\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (4)$$

is the stiffness tensor in Voigt's notation, K_v - bulk modulus and *G* – shear modulus of the material.

The application of the weak formulation and Galerkin weighting functions leads to the classical dynamic elasticity equation for a finite element as

$$\begin{aligned} &[\mathbf{M}^e]\{\ddot{\mathbf{U}}^e\}+[\mathbf{C}^e]\{\dot{\mathbf{U}}^e\}+[\mathbf{K}^e]\{\mathbf{U}^e\} \\ &\qquad =\{\mathbf{R}^e(t)\}+\{\mathbf{P}^e(t)\}+\{\mathbf{S}^e(t)\}, \qquad (3) \end{aligned}$$

where $[\mathbf{M}^e]=\rho\iint_{V_e}[\mathbf{N}]^T[\mathbf{N}]dV$, $[\mathbf{K}^e]=\int_V[\mathbf{B}]^T[\mathbf{D}][\mathbf{B}]dV$ is mass and stiffness matrices of an element, $\{\mathbf{S}^e\}=\int_S[\mathbf{N}]^T\{\mathbf{t}\}ds$, $\{\mathbf{P}^e\}=\int_{V^e}[\mathbf{N}]^T\{\mathbf{b}\}dV$ is nodal force vectors caused by surface and body loads, $\{\mathbf{R}^e\}$ is nodal vector containing the lumped forces, $[\mathbf{N}]$ is form function matrix, $[\mathbf{B}]=[\mathbf{A}][\mathbf{N}]$ is matrix defining the linear relation between strains and displacements. The structural damping forces are assumed to be very small and expressed by means of the proportional damping matrix $[\mathbf{C}^e]=\alpha[\mathbf{M}^e]$. In many practical problems of ultrasonic measurement they can be neglected by assuming $\alpha = 0$. In our model the surface traction vector $\{\mathbf{t}\}$represents the acoustic pressure obtained from the diffraction model.

The inherent distortions of propagating short wave pulses in discrete meshes, sometimes physically interpreted as 'refractions form the nodes' require very dense meshes that make transient short waves and wave pulses simulation computations complex and demanding huge computational resources. The main difficulties arising in simulation of ultrasonic measurement process are caused by computational models of very large dimensionality (the smallest 2D problems of any practical value require to use models consisting of 10^6-10^7 elements) and very large number of time integration steps (inversely proportional to the linear dimension of elements).

Because of large dimensionality of the FE models representing the wave propagation domains, traditional commercial FE software is quite inefficient for solving problems of such kind. Therefore the computations have been performed by using the explicit FE code developed by us at Kaunas University of Technology for the simulation of short wave propagation in solids and fluids. The distinguishing features of the computer program are that it is implemented by using the scheme of interacting sub-domains and the combined non-diagonal mass matrices of sub-domains are used in order to increase the convergence rate of the model and enable to use fewer nodal points per wavelength.

Simulation results

The simulated system is schematically presented in Figure 2. It consists of the bipolar transducer situated at 20mm distance from the object. The object where the surface waves are supposed to be generated is 15mm thickness steel plate. The geometry and dimensions investigated bipolar transducer are shown in Figure 3a. The transducer has been excited by 2.5 period, 5MHz radio pulse.

The transducer of such configuration was selected with assumption that two opposite polarity beams were to be generated. The assumption has been confirmed by the simulation results presented in Figure 3b, where the cross-section of the ultrasonic field in the plane xOy is shown. The ultrasonic waves of the two beams create the non-uniform distribution of pressure on the surface of the object. The cross-section of the field on the surface of the plate at time instance t=13.5μs is presented in Figure 4. The positive and negative pressure pulses from both sides of the central line can be clearly seen. Near the transducer axis the acoustic pressure is almost zero. It was expected that this non-uniform pressure distribution should generate surface waves.

This dynamic non-uniform distribution of the acoustic pressure on the surface along x axis of the plate was used as input data for the finite element model. As a result, the time law and the spatial distribution of the displacement components u_y and u_x , as well as, velocity components v_y , v_x inside of the object has been obtained.

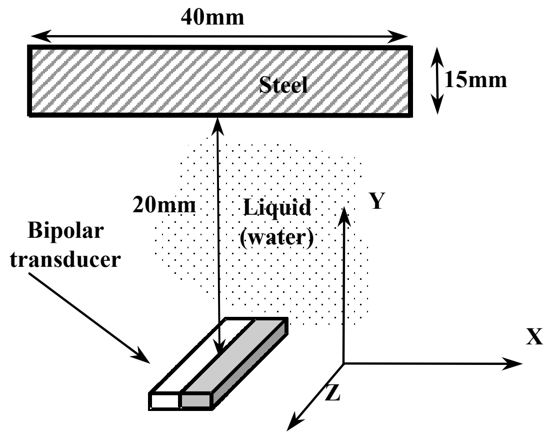

Figure 2: The geometry of simulated displacement measurement technique

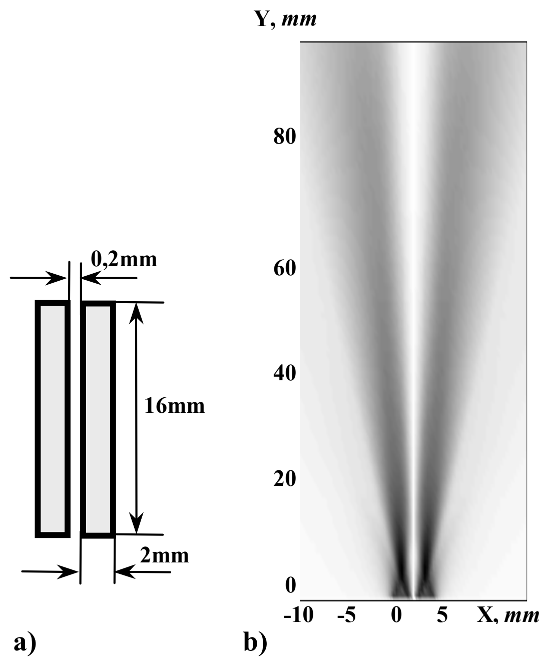

Figure 3: Dimensions of bipolar transducer (a) and structure of excited ultrasonic field

The obtained ultrasonic field inside of the steel plate is presented in Figure 4. Practically, from the obtained particle velocity field is quite difficult to separate the different types of waves. The longitudinal and shear waves can be separated by estimating the propagation distances and the wavelength. The wavelength of shear waves is shorter and during the same time they cover almost twice shorter distance.

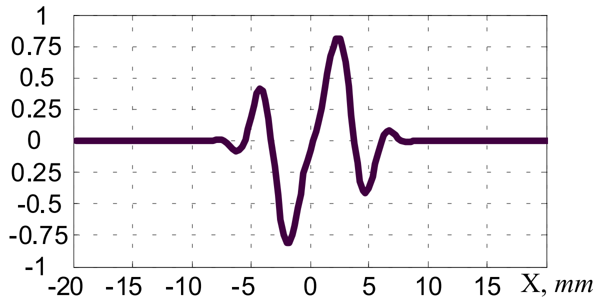

Figure 4: Distribution of relative acoustic pressure on the surface of the object

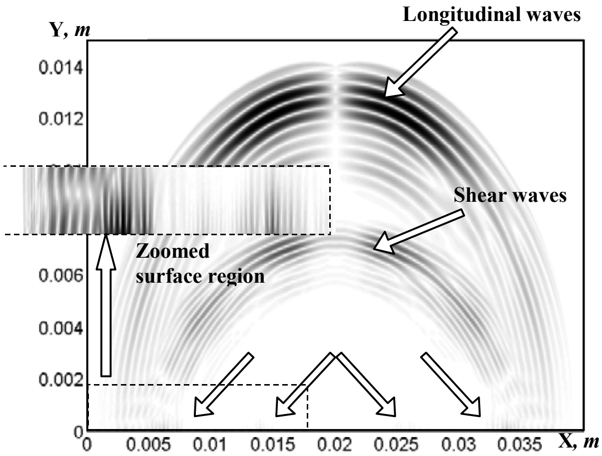

Figure 5: Distribution of modulus $v = \sqrt{v_x + v_y}$ of particle velocity inside the object at time instance t=16.2μs after transmitter excitation.

The surface waves are not so clearly expressed. They can be seen in the region close to the bottom surface y=0 of the object. In more detail they can be seen in the zoomed image, Figure 5. Especially difficult is to separate the surface wave in regions close to x=0.005m and x=0.035m where they are mixed with the patterns of longitudinal and shear waves. For the separation of the surface waves their property of fast decay with the depth was used. In Figure 6 the amplitude of the particle velocity on the surface and at the depth 0.6mm is presented. The depth 0.6mm corresponds approximately to the wavelength of the expected surface waves. At such a depth their amplitude should be essentially suppressed. As it can be seen from the Figure 6b there are four signals the amplitude of which is essentially less in comparison with the amplitude measured on the boundary of the object (Figure 6a).

Conclusions

The mixed diffraction-based and finite element model has been developed for the investigation of complicated ultrasonic measurement cases based on surface and plate wave generation.

The investigation has shown that the transducers with asymmetric field, such as bipolar transducers, could be used for the generation of surface waves and, as consequence, for the displacement measurement.

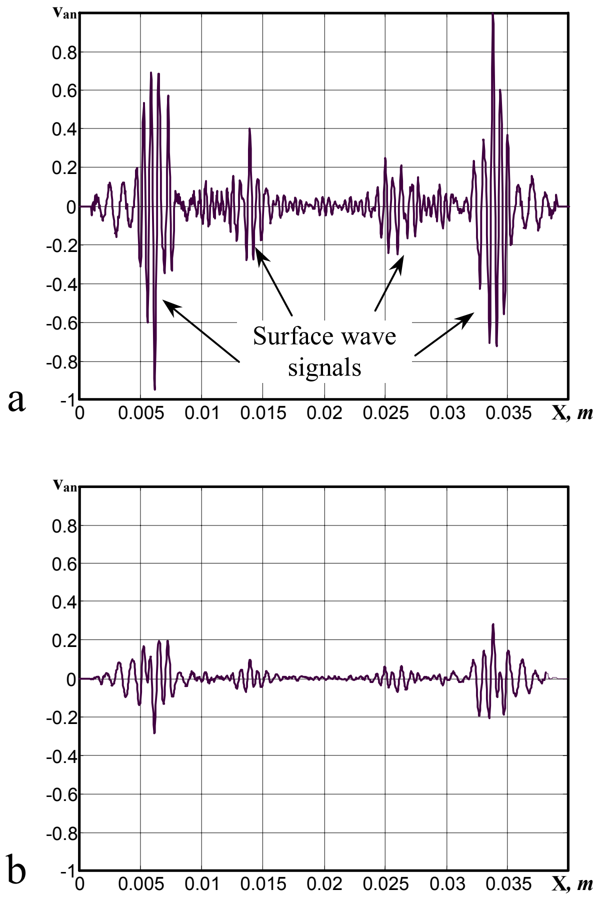

Figure 6: Distribution of particle velocity component v_y on the surface of the object (a) and 0.6mm below the surface (b) at time instance t=16.2μs

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